Rossmoyne Senior High School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words ______________________________________

Teacher's name: Mr. Fletcher Mr. Freer Mr. Kigodi Ms. Leonard Mr. Tanday

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

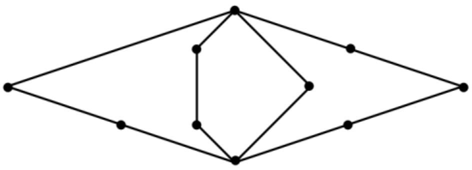
(a) In a group of five people it was known that Eli was older than Cara, Dan and Fi; Dan was older than Cara and Fi; Gus was older than Eli and Fi; and Cara was older than Fi.

(i) Represent this set of age relationships as a digraph. (2 marks)

(ii) State the number of arcs in the digraph. (1 mark)

(iii) List the five people in order of age, starting with the youngest. (1 mark)

(b) Graph H is shown below.



Let t and p be the number of edges in the longest open trail and shortest closed path contained in H respectively. State the values of t and p, given that $t > 0$ and $p > 0$. (2 marks)

Question 10 **(8 marks)**

The following data shows the blood haemoglobin (H) levels and packed cell volumes (V) of 10 blood bank donors.

H	15.5	13.6	13.8	12.9	12.5	12.3	14.8	13.1	16.1	16.4
V	0.45	0.42	0.44	0.42	0.41	0.39	0.43	0.41	0.45	0.47

(a) Graph the data on your calculator and describe features of the graph that suggest the presence of a strong and positive linear association between H and V. (2 marks)

(b) Determine the equation of the least-squares line that models the relationship between H and V, where H is the explanatory variable. (2 marks)

(c) Calculate the correlation coefficient between H and V. (1 mark)

(d) What percentage of the variation in V can be explained by the variation in H? (1 mark)

(e) Predict the packed cell volume of a donor with a blood haemoglobin level of 13.5. (1 mark)

(f) Describe a potential danger associated with using the least-squares line to predict a packed cell volume from a blood haemoglobin level of 18. (1 mark)

Question 11 **(7 marks)**

A company bought and installed a new computer system with an initial value of $\$36\ 960$. For accounting purposes, the value of the system decreased by $\$2\ 310$ each year.

(a) Calculate the value of the system after one year. (1 mark)

(b) Determine a recursive equation for V_n, the value of the system after n years. (2 marks)

(c) Determine

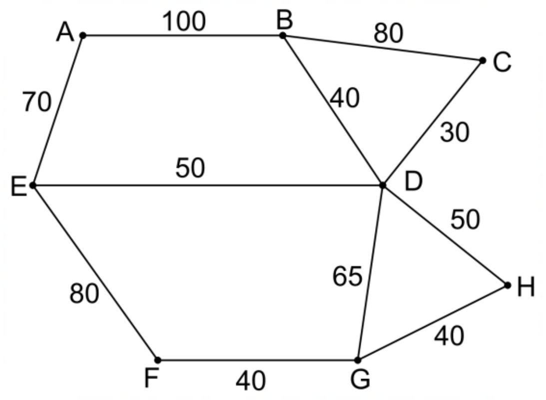
(i) the value of the system after 10 years. (1 mark)

(ii) the number of years for the value of the system to become nothing. (1 mark)

(d) Determine the decrease in time taken for the system to become worthless if its value decreased by $\$3\ 080$ each year instead of $\$2\ 310$. (2 marks)

Question 12 **(8 marks)**

This network represents a road system connecting country towns. The distances are in kilometres.



(a) John starts at A, travels to B, then continues to each town, returning eventually to A.

(i) If he visits each town only once, what is the name given to this circuit? (1 mark)

(ii) Determine the length of this circuit. (1 mark)

(b) (i) If John starts at A, but knows that C is not reachable directly from D, what path can he take to visit each town only once, if he doesn't return to A? (2 marks)

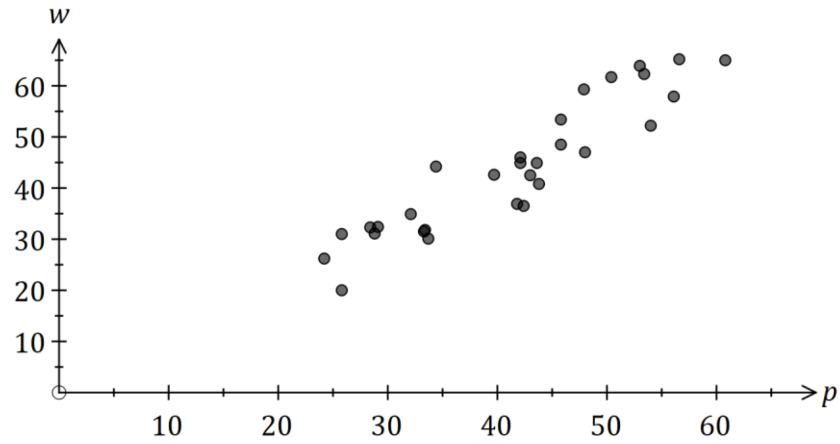
(ii) What is the name given to such a path? (1 mark)

(iii) What is the length of this path? (1 mark)

(c) While John is at H, he receives a call from his boss at A, asking John to get to A as soon as possible.
If John can average 100 km/hr, how long will the trip take, correct to the nearest minute? (2 marks)

Question 13 **(7 marks)**

The scatterplot below shows the marks scored by 30 students in their practical (p) and written (w) exams. The practical and written exams were both marked out of 85.



The equation of the least-squares line for the data is $w = 1.15p - 3.8$.

(a) It was found that 86% of the variation in w could be explained by the variation in p. Determine the correlation coefficient r_{pw}. (1 mark)

(b) Interpret the slope of the least-squares line. (2 marks)

(c) Dee and Eve were absent for the written exam, but it was known that their marks in the practical exam were 19 and 52 respectively. Predict their written exam marks and explain how reliable each prediction is. (4 marks)

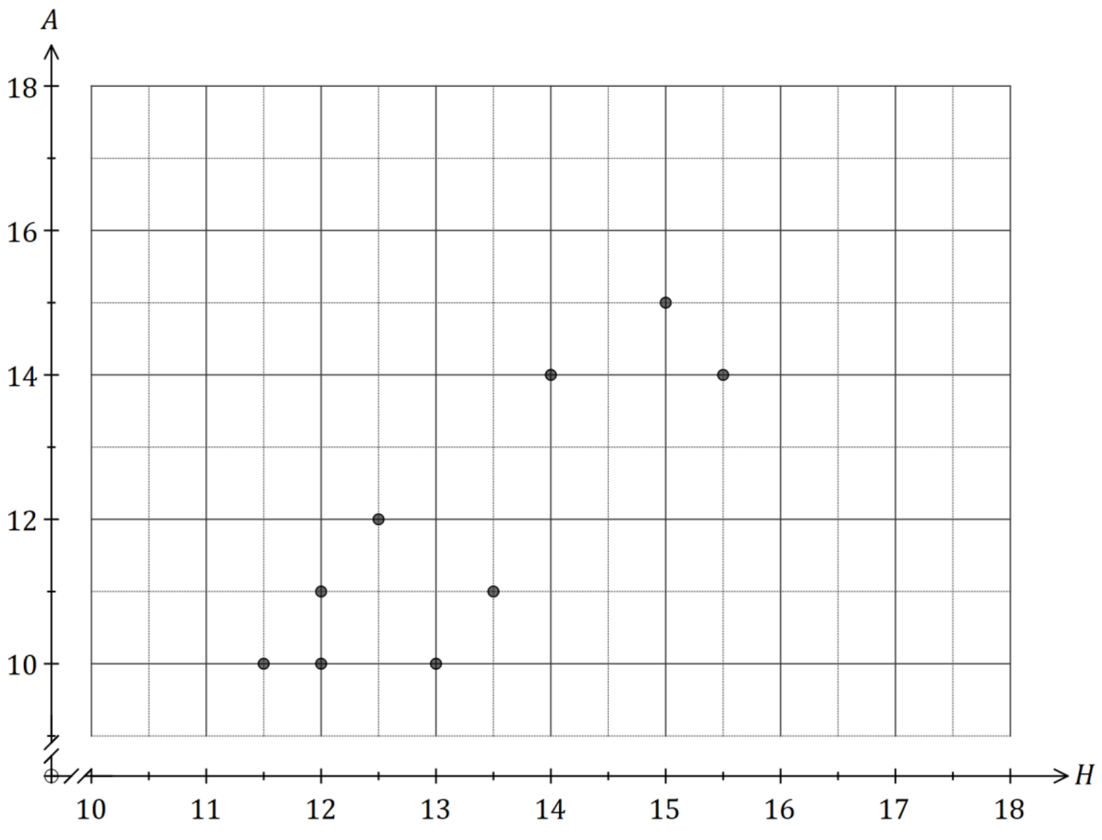
Question 14 **(11 marks)**

A researcher obtained the following data whilst investigating whether it is possible to reliably predict a child's reading ability (A, on a numerical scale of 1 to 25) from their hand span (H, cm).

Child	B	C	D	E	F	G	J	K	L	M	N	P
H	11.5	12.5	16.5	15.0	14.0	13.5	12.0	13.0	17.0	15.5	14.5	12.0
A	10	12	17	15	14	11	11	10	15	14	16	10

(a) State the response variable for this investigation. (1 mark)

(b) Add the three missing data points to the scatterplot below. (2 marks)



(c) Determine the correlation coefficient between the two variables. (1 mark)

(d) Using the scatterplot from (b) and the correlation coefficient from (c), the researcher was satisfied that a linear associated existed between A and H. Explain why they reached this conclusion. (2 marks)

The researcher then discovered that the children labelled B, C, G, J, K and P were all in Year 4 and the remainder in Year 7.

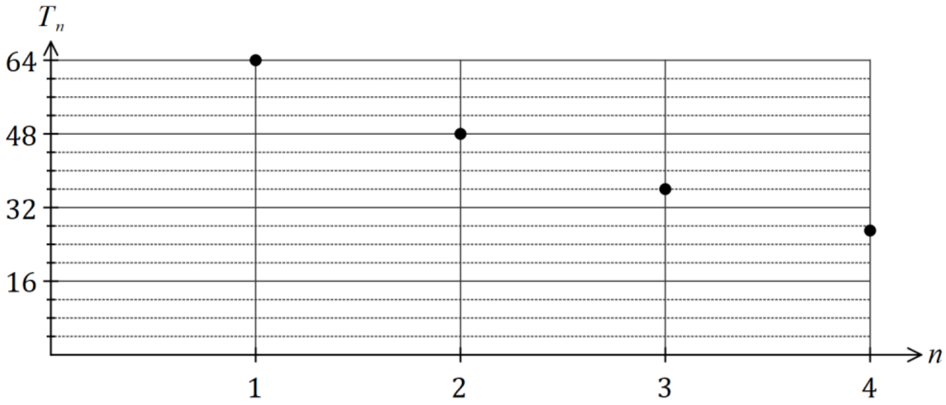
(e) Circle the Year 4 children on the graph. (1 mark)

(f) Calculate the correlation coefficient between A and H for the Year 4 children only. (1 mark)

(g) Identify a non-causal explanation for the conclusion reached by the researcher in (d) and explain how this new information affects that conclusion. (3 marks)

Question 15 **(6 marks)**

A piledriver is hammering a pile into the ground. The graph below shows the distance T_n (in cm) the pile moves into the ground on the n^{th} hit of the piledriver.



The values of T_n form a geometric sequence.

(a) Use information from the graph to determine the common ratio for the sequence. (1 mark)

(b) Write a recursive equation to generate the values of T_n. (2 marks)

(c) Write a rule to generate values of T_n. (1 mark)

(d) Determine

(i) the distance the pile moves into the ground on the tenth hit of the piledriver. (1 mark)

(ii) on which hit the pile first moves less than one mm into the ground. (1 mark)

Question 16 **(9 marks)**

A study categorized the weight of hospitalised children as underweight, normal, overweight or obese. The numbers of children in each category are shown by gender in the table below.

	Underweight	Normal	Overweight	Obese
Male	25	142	44	15
Female	16	155	67	27

(a) An obese child is randomly chosen from the study. If possible, explain whether they are more likely to be a boy or a girl. If not possible, explain your reasoning. (2 marks)

(b) What percentage of the girls in the study were classified as overweight? (2 marks)

(c) Complete the table of **row** percentages below to the nearest whole number. (3 marks)

(%)	Underweight	Normal	Overweight	Obese
Male	11			
Female				10

(d) Does the table of row percentages suggest the presence of an association between the categorical variables? Justify your answer. (2 marks)

Question 17 **(7 marks)**

A water tank is initially empty. At the start of each hour, 120 L of water is quickly poured into the tank but during the following hour, 15% of all the water in the tank leaks out.

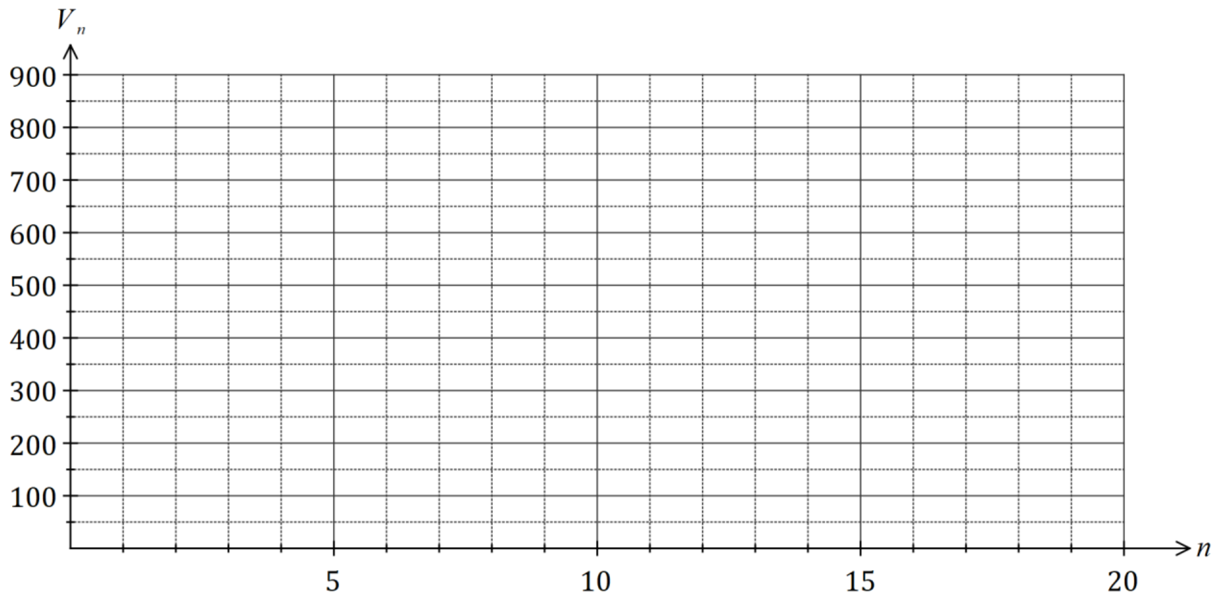
This situation can be modelled by the recurrence relation $V_{n+1} = 0.85V_n + 120,\ V_0 = 120$, where V_n is the volume of water in the tank, in litres, at the start of the n^{th} hour.

(a) Complete the table below, giving volumes to the nearest litre. (2 marks)

n	0	5	10	15	20
V_n					774

(b) At the start of which hour does the tank first hold at least 798 L? (1 mark)

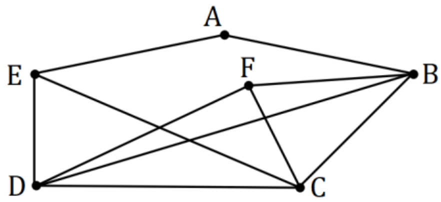
(c) Plot the points from the table on the axes below. (2 marks)



(d) The tank has a maximum capacity of 850 L. If possible, determine the least number of hours since filling commenced that the tank will start to overflow. If not possible, explain why not. (2 marks)

Question 18 **(7 marks)**

Each vertex on the graph below represents an airport and an edge between two airports indicates that an airline has a direct flight, in both directions, between the airports.



(a) Redraw the graph to clearly show that it is planar. (1 mark)

(b) **Demonstrate** that the graph satisfies Euler's formula. (2 marks)

In order to check in-flight catering quality, an airline manager plans to leave airport A, travel on at least one flight between the 10 pairs of airports and then return to A. The manager does not use any other mode of transport between airports.

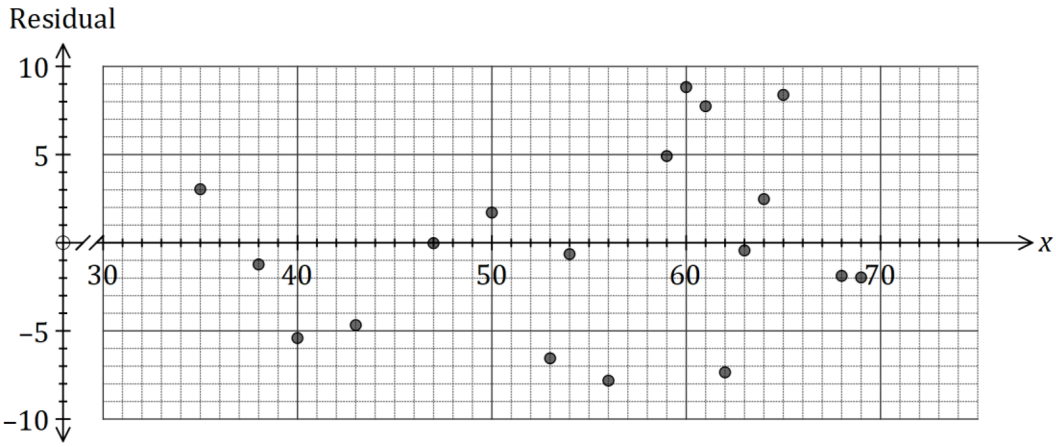
(c) Determine the minimum number of flights the manager must take and list, in order, the airports visited. (2 marks)

(d) Another manager, based at a different airport, claimed they could carry out a quality check with fewer flights, but only if they started at their airport and finished at another airport. Comment on this claim. (2 marks)

Question 19 **(6 marks)**

A statistician wants to check whether a linear model is appropriate for a bivariate data set they are analysing. The least-squares line to model the linear relationship is $y = 2.09x + 8.88$ and the correlation coefficient between the variables is very strong.

The residual plot using the linear model is shown below for all but two of the data points.



(a) Calculate the residuals for the missing points $(45, 111)$ and $(72, 152)$ and plot them on the graph above. (4 marks)

(b) Use the residual plot to explain whether fitting a linear model to the data is appropriate. (2 marks)

Question 20 **(8 marks)**

(a) An investor has $\$2\ 520$ in an account. One month later, and at the start of each subsequent month, a deposit of $\$95$ is added to the account. Interest, calculated as 0.38% of the balance at the start of the month, is added to the account just before each deposit is made.

The account balance after n deposits is T_n, and can be modelled by the recurrence relation $T_{n+1} = 1.0038T_n + 95,\ T_0 = 2520$.

(i) Determine the balance in the account after 7 deposits have been made. (1 mark)

(ii) After how many deposits does the balance of the account first exceed $\$5\ 000$ and what is the balance of the account at that time? (2 marks)

(b) The investor also has $\$385$ in another account. One week later, and at the start of each subsequent week, a deposit of $\$9.75$ is added to the account. Interest, calculated as 0.072% of the balance at the start of the week, is added to the account just **before** each deposit is made.

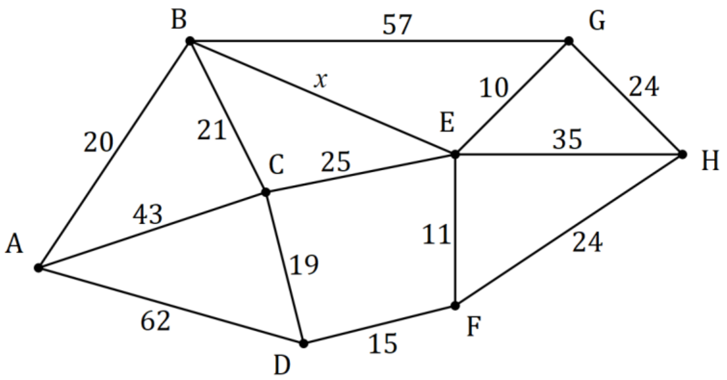
(i) Write a recurrence relation to model the balance of this account after n deposits. (3 marks)

(ii) Determine the balance in this account after 52 deposits have been made. (1 mark)

(iii) By considering the total deposits made, or otherwise, determine the total interest added to this account after 52 deposits have been made. (1 mark)

Question 21 **(8 marks)**

The vertices below represent 8 computers in a network and the weights on each edge represent the time, in milliseconds, for a signal to be sent directly between connected computers.



(a) Given that $x = 48$, determine the path required and the time taken to send a signal in the least time between

(i) B and G. (2 marks)

(ii) A and E. (2 marks)

(iii) A and H. (2 marks)

(b) Determine the largest value of x, to the nearest millisecond, to ensure that the fastest route to send a signal between A and H will pass through E. Justify your answer. (2 marks)

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________

Supplementary page

Question number: _________

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